ELIZADE UNIVERSITY, ILARA-MOKIN, ONDO STATE

DEPARTMENT OF MECHANICAL ENGINEERING

MEE 403: MECHANICAL VIBRATIONS – Exam 1st Sem: Tue 4th April 2017

INSTRUCTIONS:

a) Answer *ANY FIVE* questions

b) Make clear properly labeled sketches where required

c) You can use the Formula document given to you in class provided you have not written additional info on it

d) For calculations, you are advised to first state the steps you would use to solve the problem

Question 1

(i) Use the SHM of a simple pendulum to explain the energy method for deriving the ODE for a SDOF free undamped vibration

(ii) Write expression for the natural frequency of the following undamped systems stating the quantities involved (a) torsional vibration of a shaft-disc system, (b) transverse vibration of a cantilever beam with mass m at its free end

(iii) Define logarithmic decrement, δ, for a free damped vibration. Show that for damping ratio of $\zeta << 1.0$, $\delta \approx 2\pi\zeta$.

(iv) What are the stages involved in tackling practical problems of vibration? Use a vehicle, machine or structure to illustrate your answer

(v) Explain the procedure for measuring vibration, stating the parameters measured and the types of probes for each parameter

(vi) Describe two methods of vibration isolation using sketches for illustration. How can you determine the effectiveness of an isolation system and improve on it where necessary?

Question 2

(i) For the damped base excitation shown in Figure 1 below, r is ratio of frequencies and T is the Transmissibility. (a) Sketch the graphs on your paper and indicate the values of increasing damping. (b) Derive the condition for $r = \sqrt{2}$.

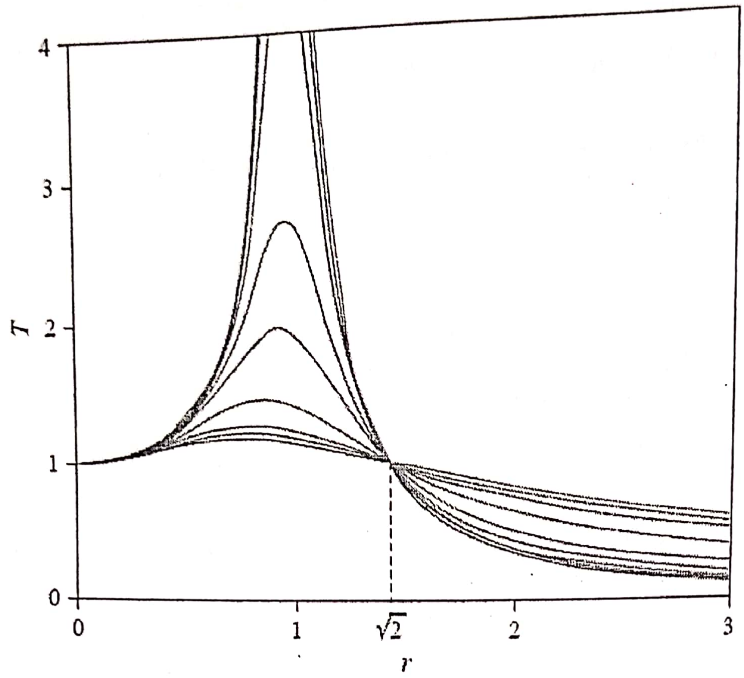


Figure 1

(ii) A 200-kg machine is attached to the end of a cantilever beam of length $L = 2.5$ m, elastic modulus $E = 200\text{x}10^9$ N/m^2, and cross-sectional moment of inertia $I = 1.8\text{x}10^{-6}$ m^4. Assuming the mass of the beam is small compared to the mass of the machine, what is the stiffness of the beam?

Question 3

A trailer of mass 1000 kg is pulled with a constant speed of 50 km/h over a bumpy road which may be modeled as a sine wave of wavelength 5 m and amplitude 50 mm (see Figure 2 below). Assume that the effective stiffness of the suspension is 350 kN/m and that the damping ratio, $\zeta = 0.5$.

(a) Determine the amplitude of the motion of the trailer

(b) Find the speed of the trailer at which this amplitude becomes a maximum (resonance).

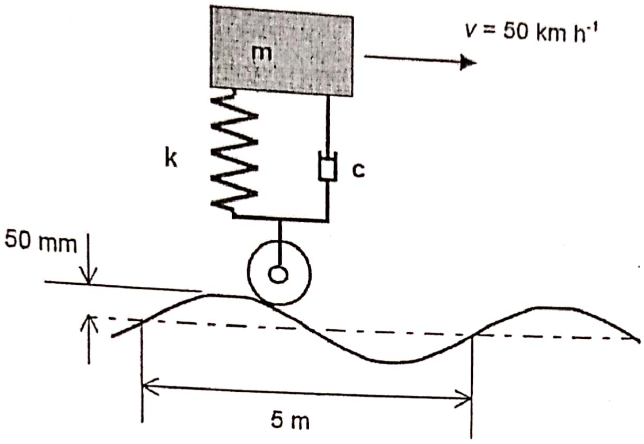


Figure 2

Question 4

Draw a sketch of the free vibration of a 2DOF undamped spring-mass system with masses m_1, m_2, and spring constants k_1, k_2, k_3 (where k_2 is for the middle spring). Show that the EOM can be written in matrix form:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Assume the solution to be of the form:

$$\mathbf{x} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} \sin \omega t$$

Let $m_1 = m$, $m_2 = 2m$, and set $k_1 = k_2 = k_3 = k$. Then derive the characteristic equation for the problem and solve for the eigenvalues, ω_i, $i = 1, 2$. Sketch the mode shapes

Question 5

For the mechanical system shown in Figure 3 below, the uniform rigid bar has mass m and is pinned at point O. For this system:

a) Find the equations of motion;

b) Identify the damping ratio and natural frequency in terms of the parameters m, c, k, and ℓ.

c) For: m = 1.50 kg, ℓ = 45 cm, c = 0.125 N/(m/s), k = 250 N/m, find the angular displacement of the bar $\theta(t)$ for the following initial conditions: $\theta(0) = 0$, $\dot{\theta}(0) = 10$ rad/s.

Assume that in the horizontal position the system is in static equilibrium and that all angles small.

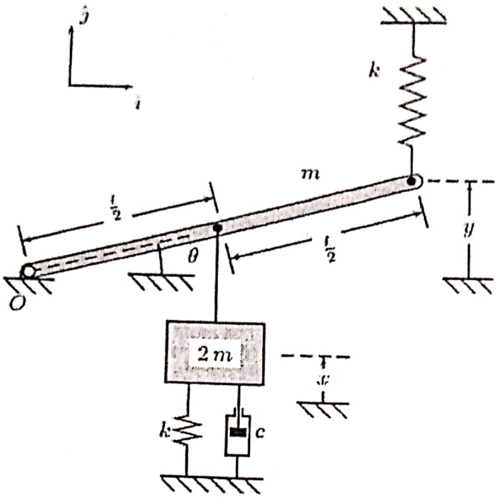


Figure 3

Question 6

For each of the vibrating systems shown in the Figures 4 (a, b, c, d) below, do the following:

i. Determine the type of vibration, i.e. longitudinal, transverse, torsional or a combination (specify)

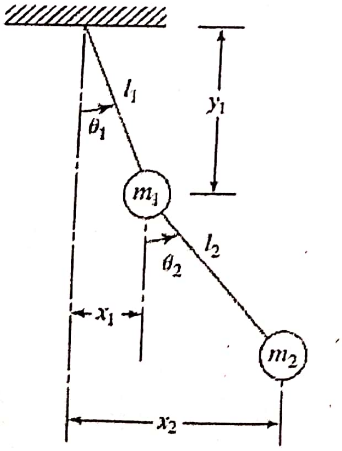
ii. Determine the number of degrees of freedom (DOF)

iii. Identify or specify suitable generalized coordinates for the number of DOF

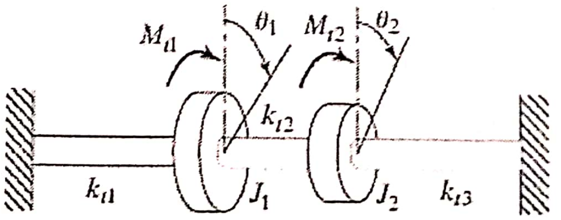
iv. Draw FBD's showing the generalized forces (i.e. **F**, **M**) and inertia forces (**ma**, **Iα**)

v. Write the differential equation(s) of motion (EOM)

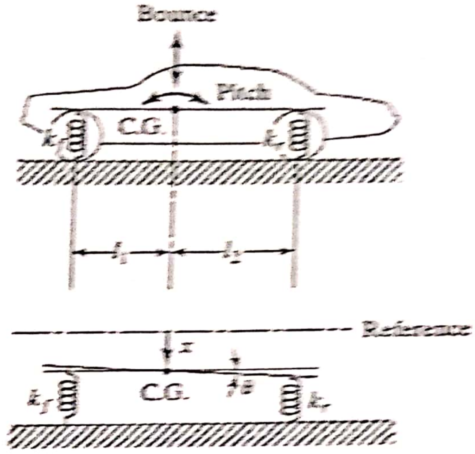
vi. State the steps or method for solving the equation(s). No need to solve the equation(s)



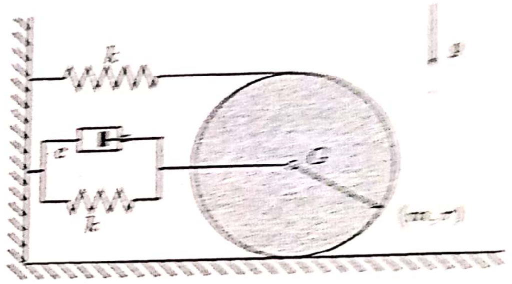
(a)



(b)



(c)



(d)

Figures 4 (a, b, c, d)

Summary: the Effects of Damping on an Unforced Mass-Spring System

Consider a mass-spring system undergoing free vibration (i.e. without a forcing function) described by the equation:

$$m\,u'' + \gamma\,u' + k\,u = 0, \qquad m > 0, \qquad k > 0.$$

The behavior of the system is determined by the magnitude of the damping coefficient γ relative to m and k.

1. Undamped system (when $\gamma = 0$)

   Displacement: $u(t) = C_1 \cos \omega_0 t + C_2 \sin \omega_0 t$

   Oscillation: Yes, periodic (at natural frequency $\omega_0 = \sqrt{\frac{k}{m}}$)

   Notes: Steady oscillation with constant amplitude $R = \sqrt{C_1^2 + C_2^2}$.

2. Underdamped system (when $0 < \gamma^2 < 4mk$)

   Displacement: $u(t) = C_1 e^{\lambda t} \cos \mu t + C_2 e^{\lambda t} \sin \mu t$

   Oscillation: Yes, quasi-periodic (at quasi-frequency μ)
   Notes: Exponentially-decaying oscillation

3. Critically Damped system (when $\gamma^2 = 4mk$)

   Displacement: $u(t) = C_1 e^{rt} + C_2 t e^{rt}$

   Oscillation: No

4. Overdamped system (when $\gamma^2 > 4mk$)

   Displacement: $u(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$

   Oscillation: No

Damped free vibration of SDOF system

- Define the critical damping coefficient c_c as that value of c that makes the radical equal to zero,

$$c_c = 2m\sqrt{\frac{k}{m}} = 2m\omega_n$$

- Define the damping factor as:

$$\xi = \frac{c}{c_c} = \frac{c}{2m\omega_n}$$

- Introducing the above equation into

$$s_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

- We find:

$$s_{1,2} = \left(-\xi \pm \sqrt{\xi^2 - 1}\right)\omega_n$$

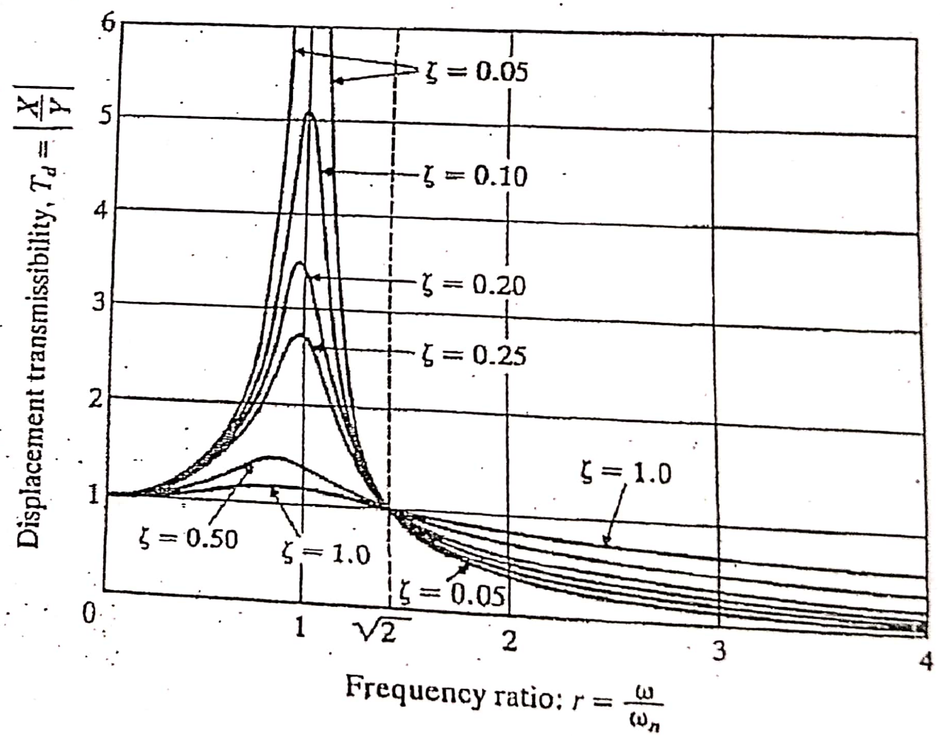
- Then the solution can be written as:

$$x(t) = Ae^{\left(-\zeta+\sqrt{\zeta^2-1}\right)\omega_n t} + Be^{\left(-\zeta-\sqrt{\zeta^2-1}\right)\omega_n t}$$

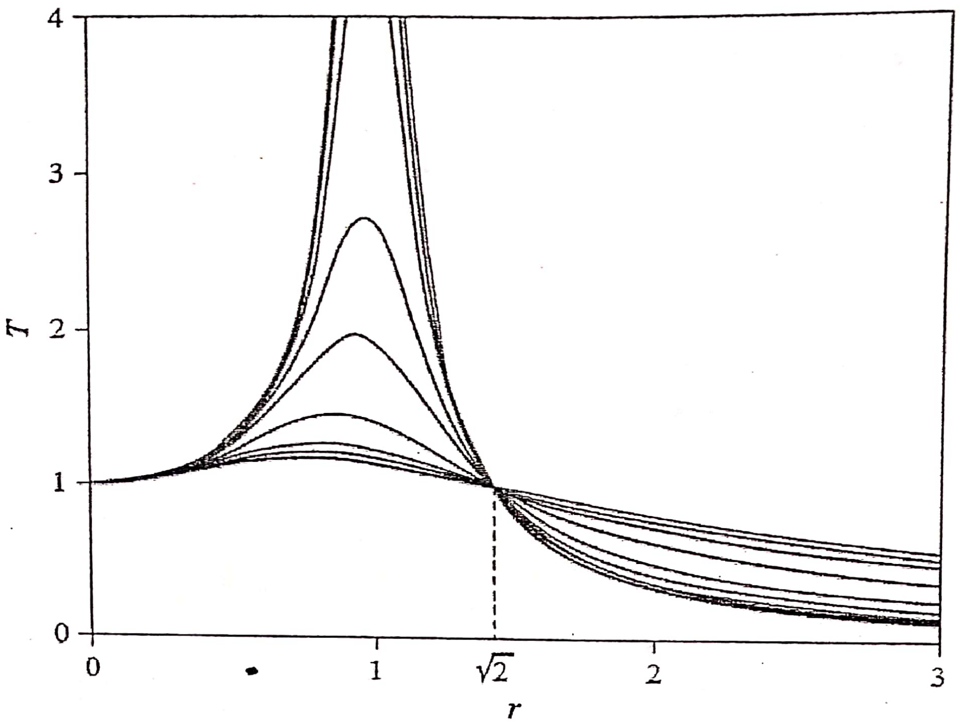
Base excited systems: absolute motion

- In nondimensional form $\frac{X_o}{Y_o} = \sqrt{\frac{1+(2\xi r)^2}{(1-r^2)^2+(2\xi r)^2}}$ $= T(r, \xi)$, transmis

- The gain function for the absolute displacement for the base-excited system is shown in the figure.



Transmissibility ratio, T



$$r_{max} = \frac{1}{2\zeta}\left(\sqrt{1 + 8\zeta^2} - 1\right)^{1/2}$$

$$T_{max} = 4\zeta^2\left[\frac{\sqrt{1 + 8\zeta^2}}{2 + 16\zeta^2 + (16\zeta^4 - 8\zeta^2 - 2)\sqrt{1 + 8\zeta^2}}\right.$$

$T(\sqrt{2}, \zeta) = 1$, independent of the value of ζ.

For $r < \sqrt{2}$, $T(r, \zeta)$ is larger for smaller values of ζ. However, for $r > \sqrt{2}$, $T(r, \zeta)$ is smaller for smaller values of ζ.

For all values of ζ, $T(r, \zeta)$ is less than one when and only when $r > \sqrt{2}$.